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## Original Research Paper

# Convection induced by vibrating rod in fine-powder bed



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## ABSTRACT

In this study, vibration-induced convection was studied experimentally using a fine powder with a mass median particle diameter of 8  $\mu\text{m}$ . A cylindrical rod arranged vertically in a powder container was vibrated horizontally with simple harmonic motion at a frequency of 300 Hz using a piezoelectric vibrator. For a vibration amplitude of 10  $\mu\text{m}$ , particles around the cylindrical rod were consolidated to a certain extent due to gravity; however, for a vibration amplitude of 70  $\mu\text{m}$  or more, a pair of convection rolls formed on both sides of the vibrating rod. The strength of the convection was quantified from the particle velocity distribution in the convection rolls, and the relationship between the convection strength and vibration amplitude was elucidated. In addition, the air-pressure distribution in the powder bed was measured showing that the convection strength correlates with the characteristic positive pressure, i.e., the average value of positive pressure measurements. Elliptical motion and circular motion as well as simple harmonic motion were applied to the cylindrical rod by adding two harmonic motions in directions orthogonally crossing each other with a phase difference of  $\pi/2$  rad. The convection of the particles varied according to the Lissajous trajectory of the cylindrical rod. Even for simple harmonic motion, heaps of a pair of convection cells overlapped each other. In the case of elliptical motion, the overlapping portion of the heaps became larger. In the case of circular motion, the two heaps were integrated into one circular heap, and there were no effects of the circumferential angle on the particle velocity and the characteristic positive pressure.

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## 1. Introduction

When shaking a container filled with granular material, the solid content can be fluidized owing to the loss of the force balance acting on each solid. As it is known that the vibration-induced fluidization results in unusual behaviors such as convection, segregation, and bubbling, a number of studies have been conducted to elucidate the mechanism of these phenomena under consideration [1–8]. Convection, in particular, has been studied for a long time. In 1831, Faraday's [9] report on the peculiar arrangement and motion of the heaps formed by particles lying on a vibrating surface was a pioneering work in the field. Evesque and Rajchenbach [10] and Laroche et al. [11] analyzed the effect of vibration acceleration on the heap formed by convective motion. Taguchi [12] and Gallas et al. [13] reproduced the convective motion in a numerical simulation and noted that the side wall of the container causes the convection. Aoki et al. [14,15] experimentally demonstrated that multiple pairs of convection rolls are formed by changing the con-

dition of the vibration. Although vibration has generally been applied to containers in the vertical direction, Liffman et al. [16], Tennakoon et al. [17], and Medved et al. [18] studied convection in a horizontally vibrated granular material. Tai and Hsiau [19] quantified the strength of the convection, and Lu and Hsiau [20] analyzed the mixing in a vibrated granular bed while considering the effect of diffusion and convection and using a three-dimensional discrete element method. In addition, the effects of various parameters on the convection were studied, e.g. the effects of the tilted side walls of a container [21], bed height [22], and non-spherical particles [23] on the formation of the convection cells. In recent years, the mechanism of the convection was also studied [24,25]. Most research on vibration-induced convection, however, has focused on larger particles having higher flowability. When using smaller particles, the interparticle adhesive forces such as the van der Waals force, electrostatic force, and liquid bridge force are greater than gravity [26,27]; thus, the movement of the particles is greatly restrained and it is difficult to fluidize these small particles.

Matsusaka et al. [28] and Mizutani et al. [29] reported that a fine powder with a mass median particle diameter of 8  $\mu\text{m}$

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circulates in a cylindrical container on applying a horizontal vibration of 300 Hz to it. The bubbles generated in the vibrated powder bed play an important role in this phenomenon, i.e., when the vibration is applied, the upper powder layer that has a high void fraction is consolidated and the particles move downward to a certain extent due to gravity. As the space between the particles is reduced, the air is forced out of the space and thus generates small bubbles, which coalesce and become large bubbles near the side wall vibrated in the normal direction and then move toward the top surface of the powder bed. On the other hand, the particles at the center of the powder bed and near the wall that vibrated in a tangential direction move downward; consequently, a pair of convection rolls are formed in the cylindrical container. In addition, it is noted that the bubbles generated at the bottom have a positive pressure but the air pressure in the downflows is negative, and the local pressure in the powder bed approaches the atmospheric pressure as the height increases. Furthermore, their experiments showed that a small opening arranged at the bottom of the container, where the local air pressure is negative, induces the inflow of outside air into the powder bed and forms a vigorously bubbling fluidized bed.

Once the convection rolls are induced in the powder bed, each particle is in a dynamic state; thus, adhesive forces between the particles are significantly reduced compared with those in a stationary state. As a result, the agglomeration in the powder bed will be prevented and stable convection rolls can be maintained. Even though the adhesive powder is stirred with a mechanical agitator, it is difficult to sufficiently disintegrate agglomerated particles. However, on using a vibration-induced convection technique, the powder flowability will be improved and agglomeration will be effectively prevented owing to the steady movement of the particles. This technique is also useful in preventing particles from clogging or in creating ordered mixtures. Moreover, the improvement in the powder's flowability enables the development of new applications using the particle surfaces, such as surface modification and gas–solid reactions.

The studies on vibration-induced convection have been performed by vibrating a container; however, there exists the problem that the scale of the convection is restricted to the size of the container. If convection is induced by a vibrator arranged in a powder bed without vibrating the container, the flexibility of the operation will be enhanced by arranging the vibrators at appropriate positions.

In this research, a cylindrical rod that is arranged vertically in a fine powder bed is vibrated horizontally with a simple harmonic motion using a piezoelectric vibrator, and the behavior of the particles is analyzed. In addition, the air-pressure distribution in the powder bed is measured to investigate the relationship between the particle behavior and the air pressure. Furthermore, to elucidate the effect of motion variation on the particle behavior, elliptical motion and circular motion are applied to the cylindrical rod by adding two harmonic motions generated by two piezoelectric vibrators in directions orthogonally crossing each other.

## 2. Experiments

### 2.1. System with a simple harmonic motion mechanism

Fig. 1 shows a schematic of the experimental setup. To facilitate the observation of the behavior of the particles, the container was made of transparent acrylic plates and had a narrow rectangular shape (20-mm wide, 200-mm long, and 60-mm high). The cylindrical rod with a length of 180 mm and a diameter of 16 mm, made of electrically conductive polymer (Nylatron® MC501CD R2:  $1\text{--}10^2 \Omega\text{m}$ , Quadrant Polypenco Japan Ltd) was ver-

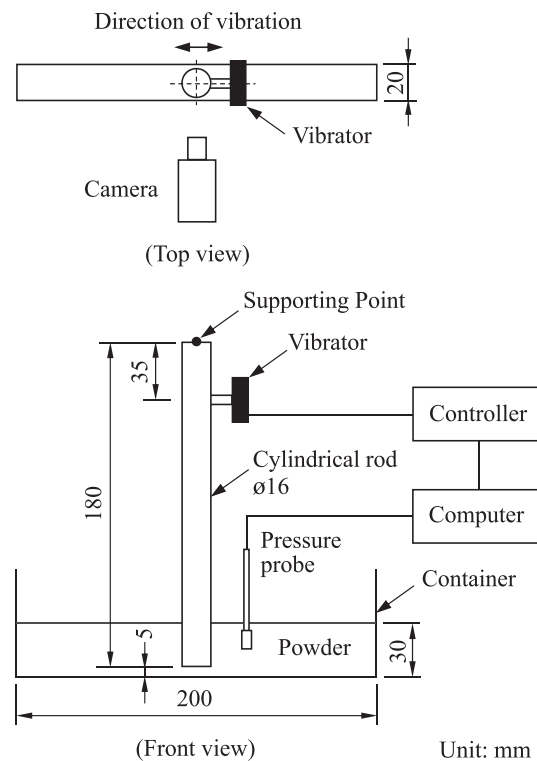


Fig. 1. System with a simple harmonic motion mechanism.

tically arranged with its top end as a fulcrum. The bottom end of the rod was set at 5 mm above the bottom plate of the container. The rod was vibrated in the longitudinal direction of the container with simple harmonic motion by a piezoelectric vibrator, which was mounted on the upper side of the rod, i.e. 35 mm from the top end, to secure a work space for filling a container with powder. A controller (VST-01, IMP. Co., Ltd.) was used to control the vibration; the frequency of the vibration was set to 300 Hz considering the resonance of the system, and the amplitude of vibration at the bottom end of the rod was set in the range of 0–120  $\mu\text{m}$ .

### 2.2. System with an elliptical or circular motion mechanism

Fig. 2 shows a schematic of the experimental system, where the bottom end of the cylindrical rod can move with an elliptical or circular motion as well as simple harmonic motion. In this system, two piezoelectric vibrators were mounted at 20 mm and 70 mm from the top end of the rod in a direction orthogonally crossing each other. The frequencies of vibration were set to 300 Hz, and each amplitude of vibration was independently controlled; the phase difference between the vibrations was set as  $\pi/2$  rad to make the bottom end of the rod perform elliptical or circular motion. A container with a square opening of 100 mm on each side was used considering the horizontal spread of the particle movement in two-dimensional vibrations.

### 2.3. Experimental procedure

The powder used was white fused alumina with a mass median diameter of 8  $\mu\text{m}$  and a particle density of 4000  $\text{kg/m}^3$ ; the particle shape was irregular. To properly evaluate the behavior of the particles under vibration and to reduce the amount of powder material, the initial bed height was set at 30 mm. To clearly observe the flow of particles in the powder bed, a small quantity of colored particles was added to the powder beforehand. The particle

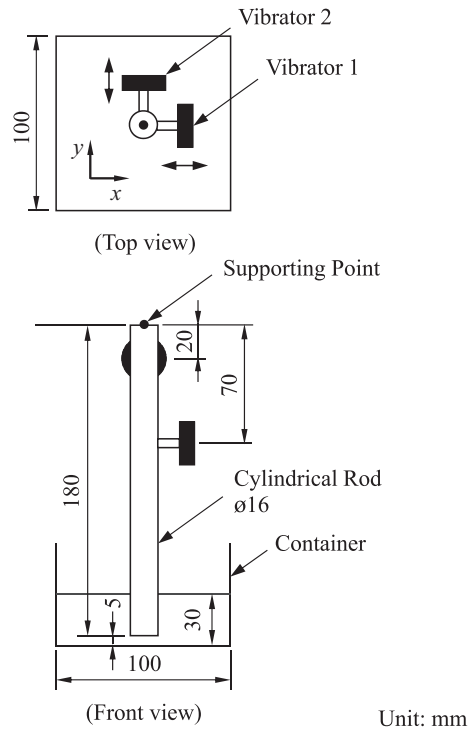


Fig. 2. System with an elliptical or circular motion mechanism.

velocity distribution in the powder bed was obtained by digital processing of video images at 30 fps (DIPP-Motion V, Ditect Corp.). Selected particles were tracked by the software; i.e. a Lagrangian approach was used. The air-pressure distribution in the powder bed was measured using a differential pressure gauge with a narrow probe (AP-47 9108570, Keyence Corp.). All the experiments were conducted at room conditions (temperature: 20–25 °C; relative humidity: 50–80%), the variation in which did not affect the experimental results.

### 3. Results and discussion

#### 3.1. Simple harmonic motion

##### 3.1.1. Observation of convection

Fig. 3 shows the snapshots of the powder bed at different amplitudes of vibration. These snapshots were taken after each steady state was maintained for 10 min. The cylindrical rod was arranged in the center of the container. To homogeneously form an initial powder bed, a small quantity of powder was repeatedly poured into the container (Fig. 3a). For a vibration amplitude of 10  $\mu\text{m}$ , the particles around the cylindrical rod were fluidized and consolidated to a certain extent due to gravity; as a result, the surface of the powder bed was compressed (Fig. 3b). As the amplitude of vibration increased, the consolidation continued and the particles on the inclined surface of the recessed portion flowed downward (Fig. 3c). For a 50- $\mu\text{m}$  vibration amplitude, a small convection roll with a radius of 10 mm appeared on the left side of the cylindrical rod (Fig. 3d). For a 70- $\mu\text{m}$  vibration amplitude, a pair of convection rolls with a radius of 10–20 mm appeared on both sides of the cylindrical rod (Fig. 3e). The particles in the left and right convection rolls were continuously moved counterclockwise and clockwise, respectively. As the vibration amplitude was 80  $\mu\text{m}$  or more, the two convection rolls became larger (Fig. 3f–h). The particle flow in the convection roll was stable, but the flow stopped immediately when the vibration was stopped.

The observation of the top surface of the powder bed using a high-speed camera at 4000 fps (FASTCAM-Mini UX100, Photron Ltd.) with a zoom lens (VSZ-10100, VS Technology Corp.) showed that agglomerated particles with a diameter of several tens to several hundreds of  $\mu\text{m}$  spouted from the gap between the powder bed and the vibrating rod. This phenomenon can be explained as follows: first, as the powder bed is consolidated by vibration, air is forced out of the space between the particles and small bubbles are generated. The bubbles coalesce near the vibrating rod and move upward; particles are then spouted with the air from the top surface of the powder bed. As mentioned in the introduction section, similar phenomena have been reported [28,29], although the experimental conditions were different from those in the present study.

Fig. 4 shows a photograph taken from the front of the powder bed, where the vibration was applied in the normal direction by changing the direction of mounting of the piezoelectric vibrator to  $\pi/2$  rad. It was found that bubbles with approximately 1-mm diameter were generated in the powder bed and moved upward, and small agglomerated particles were spouted. This phenomenon implies that the bubbles are the major cause of the particle movement and the formation of convection rolls in the powder bed.

##### 3.1.2. Analysis of convection strength

Fig. 5 shows a particle velocity vector field in the powder bed obtained at a vibration amplitude of 100  $\mu\text{m}$ . The region surrounded by a broken line in this figure represents the cylindrical rod, and the arrows indicate the particle velocity vectors. A pair of convection rolls were symmetrically formed on both sides of the vibrating rod. The centers of the convection rolls were located at  $x \approx \pm 25$  mm and  $z \approx 15$  mm. It was found that the particles in the left and right convection rolls moved counterclockwise and clockwise, respectively.

Fig. 6 shows the horizontal component of the particle velocity  $v_x$  on the vertical line through the center of the convection roll as a function of the height  $z$ . The magnitude of the velocity tends to increase with an increase in the vertical distance from the center point. To evaluate the strength of convection, Tai and Hsiao [19] proposed a convection mass flow rate. We use a similar and simple convection strength  $C$ , i.e.,

$$C = \int_{z_c}^{z_{\max}} v_x(z) dz \quad (1)$$

where  $z_{\max}$  is the powder bed height, and  $z_c$  is the center of the roll. This is because convections formed in the system are in a steady state, where mass flow rate for the upper half will be the same as that for the lower half.

Fig. 7 shows the effect of the amplitude of vibration on the convection strength  $C$ , where the error bars represent the standard error of the mean ( $n = 5$ ). After exceeding a vibration amplitude of 60  $\mu\text{m}$ , the value of  $C$  starts to rise, and greatly increases in the range from 70 to 100  $\mu\text{m}$ . After exceeding 100  $\mu\text{m}$ , the rate of increase, however, decreases. The mass flow rate of the particles can be roughly estimated by multiplying the 20-mm width of the vessel and 1200- $\text{kg}/\text{m}^3$  bulk density to the convection strength. For example, the mass flow rate is 0.5 g/s at  $C = 20 \text{ mm}^2/\text{s}$ .

##### 3.1.3. Analysis of air pressure in powder bed

Fig. 8 shows a differential air pressure distribution in the powder bed obtained at a vibration amplitude of 100  $\mu\text{m}$ . The radius of the cylindrical rod was 8 mm, and the measurement of the air pressure was in the range of  $x = 10$ –30 mm in the longitudinal direction and  $z = 0$ –25 mm in the height direction. The measurement points were arranged in a square lattice with intervals of 5 mm. The measurement at each point was repeated 5 times, and 30 average

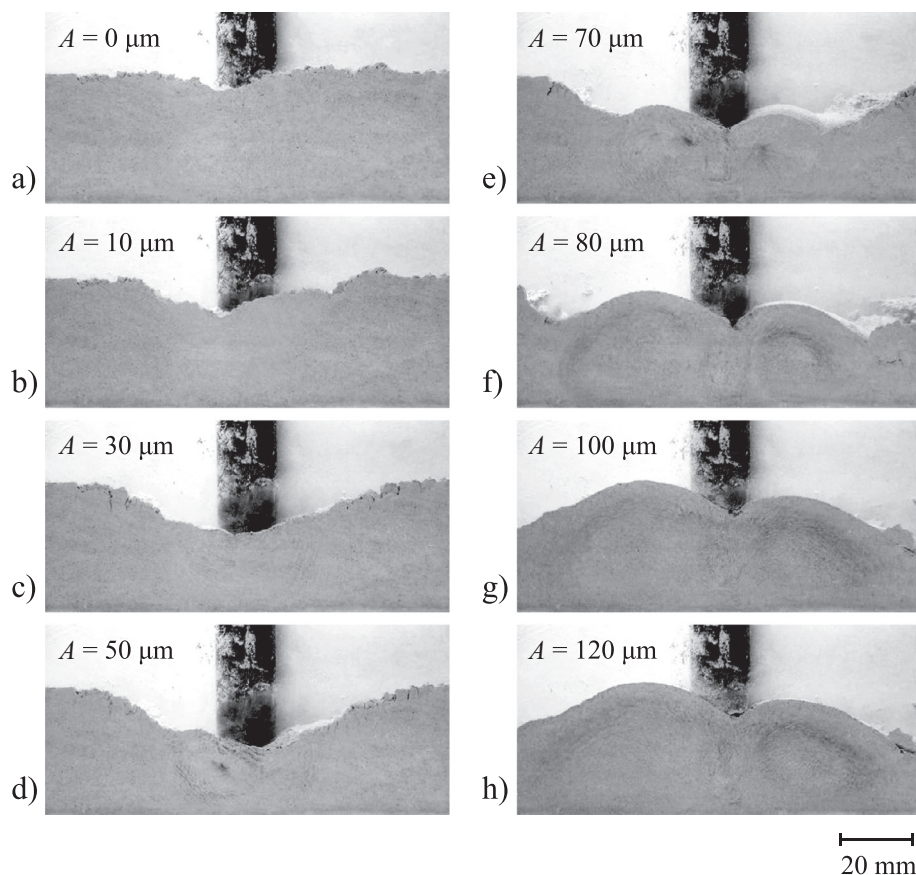


Fig. 3. Snapshots of powder bed taken at different amplitudes of vibration.

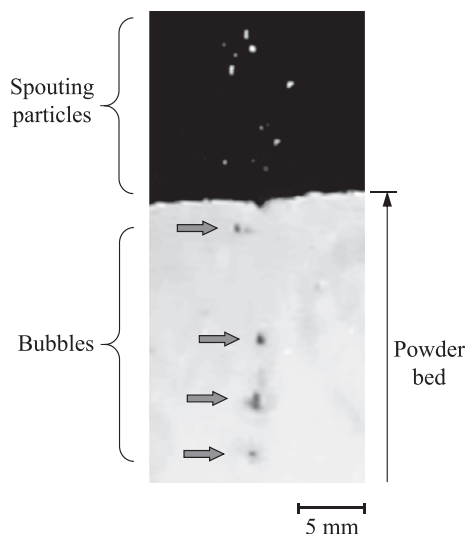


Fig. 4. Bubbles in powder bed and spouting of small agglomerated particles.

values are shown in this two-dimensional distribution. The differential air pressure in the center of the convection roll was negative (approximately  $-50$  Pa), and the region of negative pressure was spread above the center of the roll. On the other hand, the positive pressure was spread below the center of the convection roll and near the vibrating rod. The value of the positive pressure near the vibrating rod was especially large (approximately  $250$  Pa). As shown in Fig. 4, the bubbles generated in the powder bed moved upward and particles spouted from the top surface. Therefore,

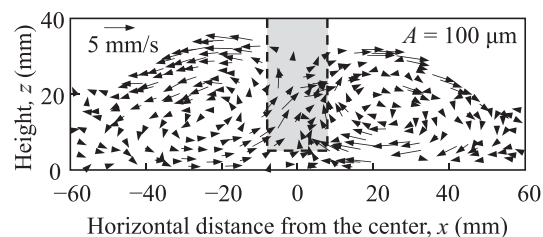


Fig. 5. Particle velocity vector field obtained by digital processing of video images.

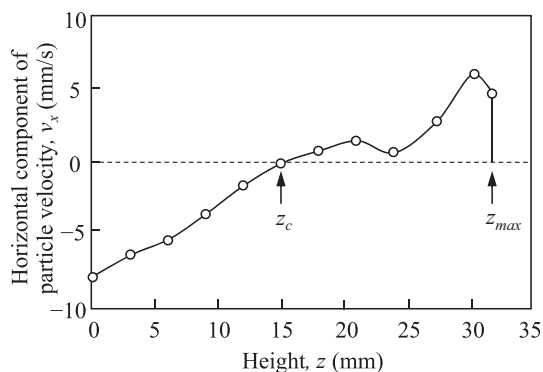


Fig. 6. Horizontal component of particle velocity as a function of height.

these experimental results also show that the bubbles with positive pressure play an important role in the convection. To evaluate the positive pressure in the powder bed at each vibration



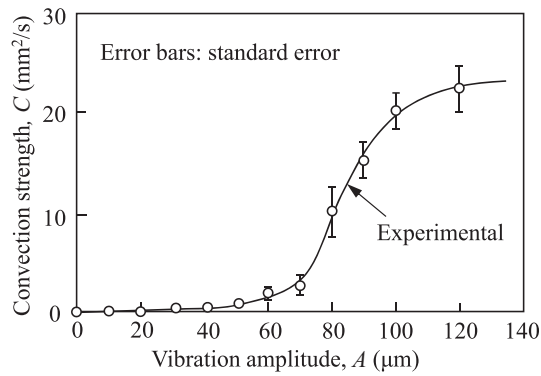


Fig. 7. Effect of vibration amplitude on convection strength.

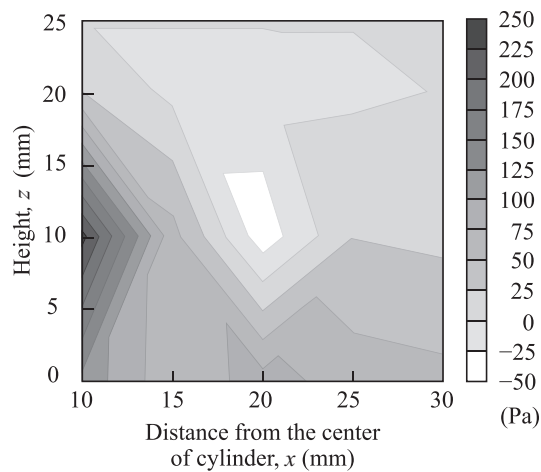


Fig. 8. Differential air pressure distribution in powder bed.

condition, we define the characteristic positive pressure  $P_p^*$  as the average value of positive pressure measurements.

Fig. 9 shows the effect of the amplitude of vibration on the characteristic positive pressure  $P_p^*$ , where the error bars represent the standard error of the mean ( $n = 5$ ). The positive pressure increased with an increase in the amplitude of vibration. In particular, the rate of increase in pressure accelerated after exceeding a vibration amplitude of 50  $\mu\text{m}$ , at which a convection roll was formed (see Fig. 3).

Fig. 10 shows the relationship between the convection strength and the characteristic positive pressure. These data are based on the experimental results shown in Figs. 7 and 9. It was found that there is a positive correlation between the two aforementioned variables. To form a convection roll, the value of  $P_p^*$  is required to

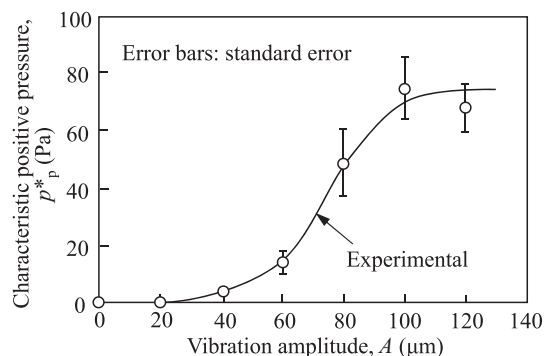


Fig. 9. Effect of vibration amplitude on characteristic positive pressure.

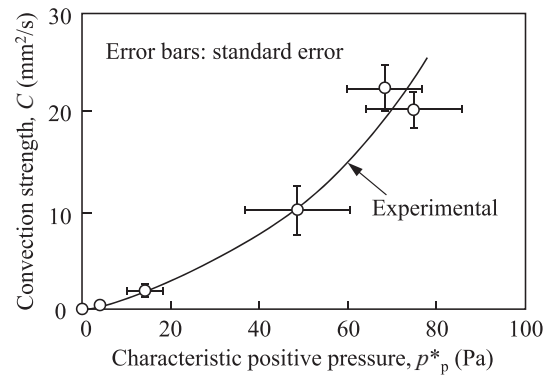


Fig. 10. Relationship between convection strength and characteristic positive pressure.

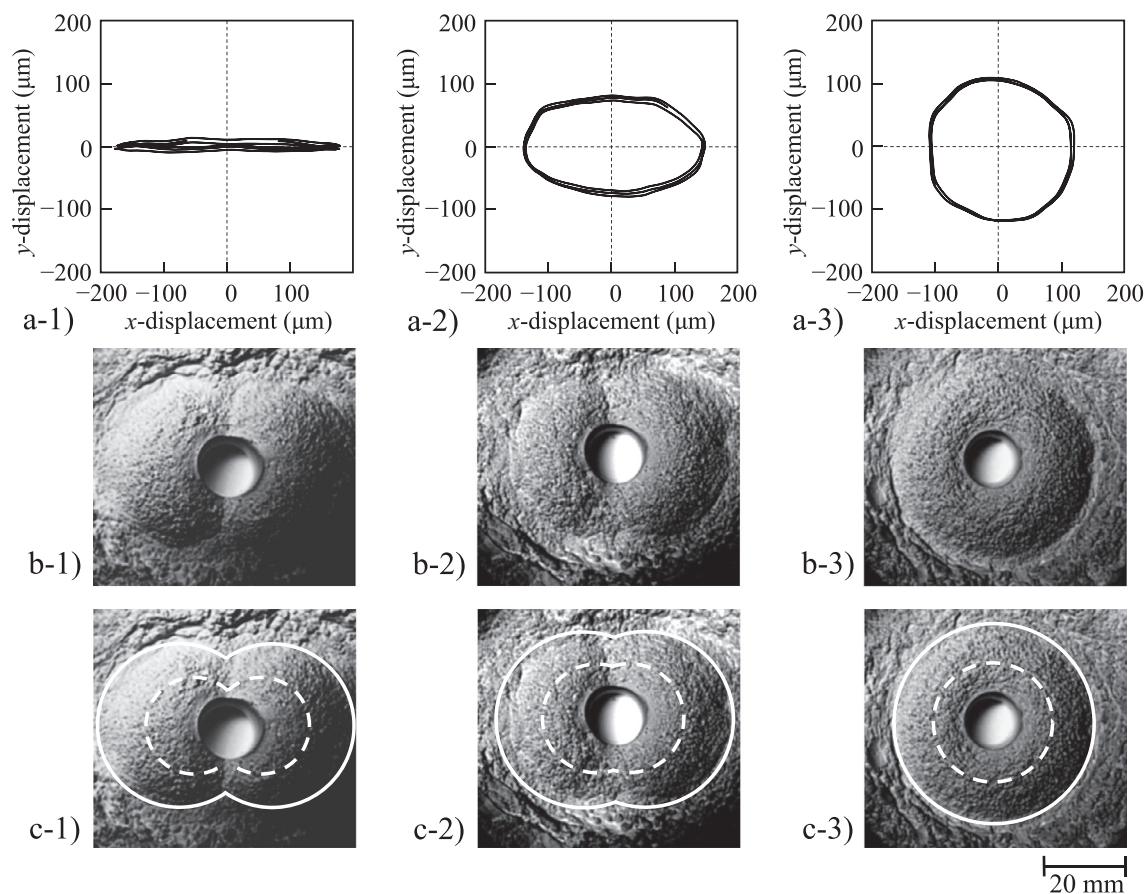
be at least 20 Pa and to sufficiently develop the convection roll,  $P_p^*$  should be larger than 60 Pa.

From the above series of experiments, both the formation of the convection rolls due to the vibrating cylindrical rod and the characteristic positive pressure in the powder bed can be explained as follows. As the particles are fluidized by applying a vibration, the powder bed is consolidated by gravity; the air is then forced out of spaces between the particles and small bubbles are generated. As the bubbles coalesce near the vibrating cylindrical rod and move to the top of the powder bed, an upward flow of particles is generated. At the top surface of the powder bed, the positive air pressure is released and the consolidated state of particles is also released; thus, the apparent particle density decreases. On the other hand, at a region away from the vibrating rod, particles are consolidated and move downward. Consequently, a pair of convection rolls are formed on both sides of the vibrating rod. In each convection roll, a particle velocity distribution and air pressure distribution are formed. As the space between particles widens, the air pressure becomes negative, while this narrowing induces a positive air pressure. The bubbles are forced out of spaces between particles, and thus, the pressure of the bubbles is positive. The characteristic positive pressure and convection strength are related to bubbles generated in the powder bed; therefore, they are correlated with each other as shown in Fig. 10.

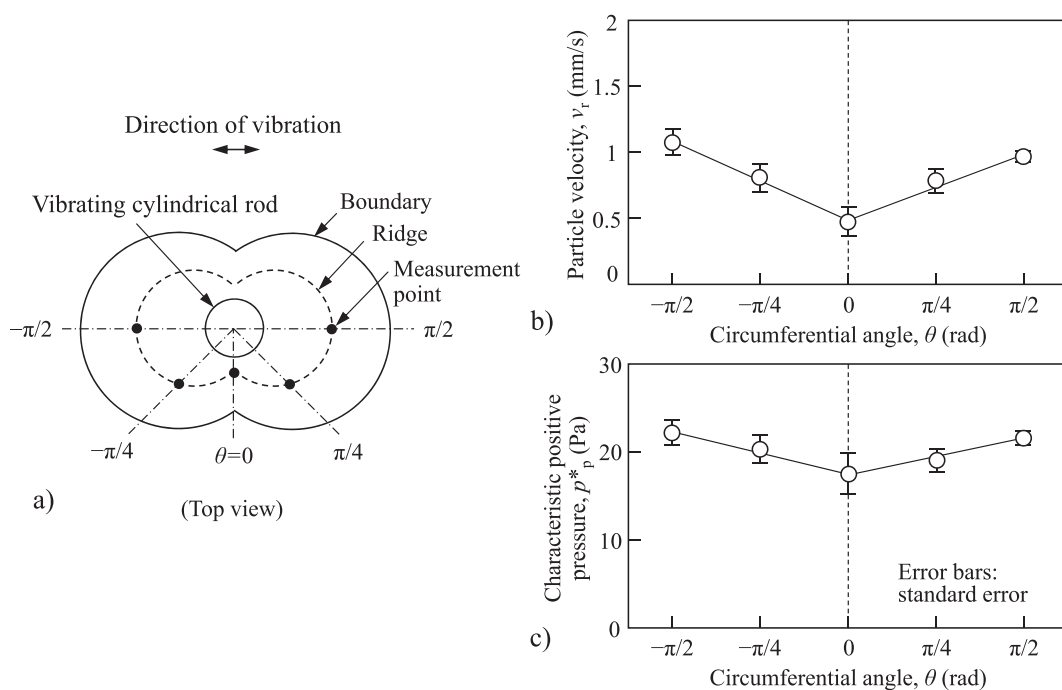
### 3.2. Comparison between simple harmonic, elliptical, and circular motions

#### 3.2.1. Observation of convection

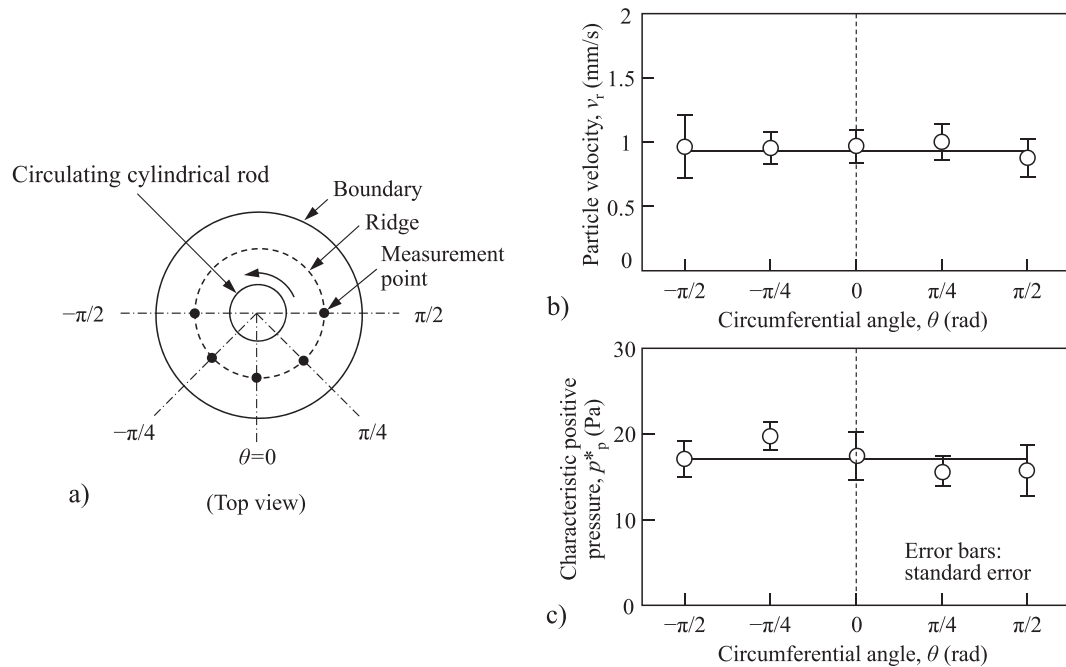
Fig. 11 shows the Lissajous trajectories of the bottom end of the cylindrical rod and the photographs of the top view of the powder beds at three different conditions (i.e., simple harmonic, elliptical, and circular motions), which can be obtained by changing the setting values of the vibration amplitude of the two piezoelectric vibrators. The data of the Lissajous trajectories were recorded at a rate of 25,000 Hz using two laser displacement sensors (LK-G30, Keyence Corp.) which were placed perpendicularly each other. These photographs were taken by removing the cylindrical rod after the experiment was conducted. A cylindrical hole that was formed by the vibrating rod was seen in the center of each photograph. For simple harmonic motion, the Lissajous curve is a straight line (Fig. 11a1), and a pair of heaps are observed on both sides of the vibrating rod (b1). The heaps were formed owing to the convection. The ridges (dashed lines) and boundaries (solid lines) of the heaps are represented by circular arcs (c1). In the system with a simple harmonic motion mechanism (see Fig. 1), the container has a narrow rectangular opening ( $20 \times 200 \text{ mm}^2$ ); thus, the movement of the particles was restricted in the minor direc-



**Fig. 11.** Effect of the motion of cylindrical rod on convection: (a) Lissajous trajectories of the bottom edge of the cylindrical rod, (b) top views of powder bed, and (c) ridges (dashed lines) and boundaries (solid lines).



**Fig. 12.** Measurement points and experimental results applied for a sinusoidal motion: (a) the top view of measurement points, (b) particle velocity, and (c) characteristic positive pressure as a function of circumferential angle.



**Fig. 13.** Measurement points and experimental results for a circular motion: (a) the top view of measurement points, (b) particle velocity, and (c) characteristic positive pressure as a function of circumferential angle.

tion. On the other hand, in the system with an elliptical motion mechanism, the container had a square-shaped opening ( $100 \times 100 \text{ mm}^2$ ). The heaps can also spread in a direction perpendicular to the direction of vibration even in simple harmonic motion, and the two heaps can overlap each other. In the case of the elliptical motion, vibrations were added to the powder bed in the minor axis direction as well as the major axis direction (a2); consequently, the overlapping portion of the heaps became larger (c2). In the case of the circular motion, the two amplitudes of vibrations were equal, and the two heaps were integrated into one circular heap (c3). Here, particles in the powder bed moved upward near the outer circumference of the cylindrical rod and moved radially on the top surface of the heap; finally, the particles moved downward. There were no particles moving in the circumferential direction for circular motion.

### 3.2.2. Analysis of particle velocity and air pressure in powder bed

Fig. 12 shows the measurement points and the experimental results of the particle velocity and characteristic positive pressure in a powder bed with a simple harmonic motion. The measurement of the particle velocity was carried out on the ridge of the powder bed ( $\theta = -\pi/2, -\pi/4, 0, \pi/4, \text{ and } \pi/2$ ), and the values were determined by digital processing of the video images. The air pressures in the powder bed were measured in the same manner as mentioned in §3.1.3. The measurement points were arranged in a square lattice at intervals of 5 mm in each sectional area at the circumferential angles of  $-\pi/2, -\pi/4, 0, \pi/4, \text{ and } \pi/2$ . The characteristic positive pressure  $P_p^*$  was quantified by the average of the positive pressure measurements. The comparison between the two experimental results in Fig. 12 shows that the effect of the circumferential angle on the particle velocity is the same as that on the characteristic positive pressure; i.e. both the values of the particle velocity and the positive pressure increase with an increase in the absolute value of the circumferential angle.

Fig. 13 shows the measurement points and the experimental results of the particle velocity and characteristic positive pressure in a powder bed with a circular motion. The measurement meth-

ods were the same as those for the simple harmonic motion mentioned above. As can be seen from the experimental results, the values of the particle velocity and the positive pressure are constant; therefore, the effects of the circumferential angle were not seen for circular motion.

## 4. Conclusions

In this study, a cylindrical rod was vertically arranged in a fine powder bed and was vibrated at 300 Hz. The particle behavior and air pressure in the powder bed at different vibration amplitudes were analyzed in detail. The results obtained are summarized as follows:

- (1) For a vibration amplitude of  $10 \mu\text{m}$ , the particles around the cylindrical rod were fluidized and consolidated to a certain extent due to gravity. In the case of a vibration amplitude of  $70 \mu\text{m}$ , a pair of convection rolls with a radius of 10–20 mm were formed on both sides of the cylindrical rod in the direction of the vibration. The particles in the two convection rolls were continuously rotated in directions opposite to each other. As the vibration amplitude was  $80 \mu\text{m}$  or more, the two convection rolls became larger.
- (2) As the convection rolls were formed, small bubbles were generated and coalesced. They moved upward near the vibrating rod and small agglomerated particles spouted with the air from the top surface of the powder bed.
- (3) To evaluate the strength of convection, the horizontal component of the particle velocity was integrated from the center of the roll to the top surface of the powder bed in the vertical direction.
- (4) The air pressure in the center of the convection roll was negative but was positive below the center and near the vibrating rod. The value of the positive pressure was especially large near the vibrating rod. The characteristic positive pressure, i.e., the average value of the positive pressure measurements in the powder bed, increased with an increase in the



amplitude of vibration. The strength of convection and the characteristic positive pressure that were caused by bubbles generated in the powder bed were correlated with each other.

- (5) The formation patterns of a pair of convection cells were changed according to the motion of the rod, such as simple harmonic, elliptical, and circular motions. Even for simple harmonic motion, the two heaps of the convection cells overlapped each other. In the case of the elliptical motion, vibrations were added to the powder bed in the minor axis direction as well as the major axis direction; consequently, the overlapping portion of the heaps became larger. In the case of the circular motion, the two heaps were integrated into one circular heap, and there were no effects of the circumferential angle on the particle velocity and the characteristic positive pressure.

## Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at <http://dx.doi.org/10.1016/j.appt.2017.07.010>.

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